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Worksheet G: Calculating probabilities



Investigation – When do the binomial and Poisson distributions give similar answers?

Task 1

Binomial	Poisson
$X \sim B(12, 0.6)$ $P(X = 4)$	$X \sim Po(7.2)$ $P(X = 4)$
$X \sim B(60, 0.6)$ $P(X = 35)$	$X \sim Po(36)$ $P(X = 35)$
$X \sim B(50, 0.2)$ $P(X = 10)$	$X \sim Po(10)$ $P(X = 10)$
$X \sim B(50, 0.05)$ $P(X = 5)$	$X \sim Po(2.5)$ $P(X = 5)$
$X \sim B(100, 0.01)$ $P(X = 5)$	$X \sim Po(2)$ $P(X = 5)$

Task 2

Compare your probabilities, what patterns do you notice? Use additional examples to investigate your conclusion.

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The number of telephone calls that arrive at a phone exchange is often modeled as a Poisson random variable. Assume that on the average there are 10 calls per hour.

- (a) What is the probability that there are exactly 5 calls in one hour?
- (b) What is the probability that there are 3 or fewer calls in one hour?
- (c) What is the probability that there are exactly 15 calls in two hours?
- (d) What is the probability that there are exactly 5 calls in 30 minutes?



Step-by-step solution

Step 1 of 6 ^

A random variable X is said to follow a Poisson distribution if it assumes only non-negative values and its probability mass function is given by:

$$P(X = x) = \begin{cases} \frac{e^{-\lambda} \lambda^x}{x!}, & x = 0, 1, 2, \dots; \lambda > 0 \\ 0, & \text{otherwise} \end{cases}$$

Here, λ is known as the parameter of the distribution.

Comment

Step 2 of 6 ^

$$\binom{n}{k} p^k q^{n-k} = \frac{1}{\sigma\sqrt{2\pi}} e^{-(\lambda-\mu)^2/2\sigma^2}, \text{ where } \mu = np \text{ and } \sigma = \sqrt{npq} \text{ (normal approximation);}$$

$$\binom{n}{k} p^k q^{n-k} = \frac{e^{-\lambda} \lambda^k}{k!}, \text{ where } \lambda = np \text{ (Poisson approximation).}$$

[The need for such approximations arises because the binomial coefficients, $\binom{n}{k} = \frac{n!}{k!(n-k)!}$, become burdensome to compute for large n and k .]

Use numerical experiments to study the "goodness" of these approximations for different values of n (e.g., 5, 10, 20) and p . Compare these approximations for k values within and outside the neighborhood $np \pm \sqrt{npq}$. Is one of the approximations better than the other? Plot and compare the approximation errors as a function of increasing n .

En el caso de la distribución binomial

• Permitir que el parámetro p se acerque a cero se relaciona con la tercera propiedad del proceso de Poisson. De hecho, si n es grande y p es cercana a 0, se puede usar la distribución de Poisson, con $\mu = np$, para aproximar probabilidades binomiales.

$$N > P \quad \mu = np \quad p = \text{PROBABILIDAD}$$

• Si p es cercana a 1, aún podemos utilizar la distribución de Poisson para aproximar probabilidades binomiales intercambiando lo que

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